Online ISSN
3007-3197
Print ISSN
3007-3189
http://amresearchreview.com/index.php/Journal/about
Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 3, Issue 5 (2025)

Disruption And Recovery Of Collective Motion In Mathematical Model Of Self-Propelled Particles Under Random Noise

[1]Israr Ahmed, [2]Mansoor Ali, [3]Sohail Ahmed Memon, [4]Abdul Majid Indher

Article Details

Keywords: Self-Propelled Particles, Stochastic Noise, Collective Motion, Phase Transition, Active Matter, Angular Noise

Israr Ahmed
Associate Professor, Department of Mathematics, Shah Abdul Latif University, Khairpur, Sinsh, Pakistan
israr.memon@salu.edu.pk
Mansoor Ali
Department of Mathematics, Shah Abdul Latif University, Khairpur, Sinsh, Pakistan
mansoor.gilal@salu.edu.pk
Sohail Ahmed Memon
Associate Professor, Department of Mathematics, Shah Abdul Latif University, Khairpur, Sinsh, Pakistan.
suhail.memon@salu,edu,pk
Abdul Majid Indher
Lecturer, Department of Basic Sciences & related studies Mehran University of Engineering and Technology, SZAB Campus Khairpur Mir's, Sindh , Pakistan.
abdulmajid@muetkhp.edu.pk

ABSTRACT

The rich collective behaviors of self-propelled particle (SPP) systems include flocking, clustering, and coherent motion. These dynamics are strongly stochastic in response, and a small variation in stochasticity may overturn the collective state and cause chaotic movement. In the present paper, the impact of random noise on the coherence of the collective motion in systems of SPPs is explored, and the circumstances under which this kind of motion may be restored. In the simulation of a variable angular noise modified Vicsek model, numerical simulations are carried out to analyse the biological processes of disruption and recovery. Critical noise level found above which order is destroyed, and shows that order may be restored when the noise level is decreased, dependent upon the density of each particle, and the length of the exposure to the noise. Phase diagrams are created in order to define the behavior of the system in various intensities and densities of noise. These results give information about the robustness and ductility of active matter systems.

DOI: Availability

INTRODUCTION

Self-propelled particles (SPPs) represent one of the basic families of active matter systems that self-propel by local energy expenditure. Examples include biological systems like bird flocks and bacterial colonies, as well as synthetic systems such as Janus particles and swarm robots (Ali M. A., 2024). The collective motion emerging in SPP systems is one of the main phenomena in this type of system, where particles are able to become aligned and move coherently because of local interactions. However, the stability of this collective behavior is influenced by stochastic perturbations, often modeled as noise (Reynolds, 2022).The dynamics of the active matter and, more specifically, self-propelled particles, has become the topic of intense interest to the physics, biology, and engineering communities (Pismen, 2021). Such systems are highly excitable and, in contrast with passive systems, have a very rich set of behaviors. Energy is consumed in active particles continuously, and this enables them to do continuous motion and have phenomena like flocking, phase separation, and clustering (Jin, 2021). The knowledge of these emergent properties of such systems is important not only to basic science but also to practical applications, including targeted drug delivery, autonomous robot design, and designing smart materials (Ali M. A., 2024).

Vicsek model is a pioneer theory of the collective movement of SPPs. In this model, the positions of the velocities of the particles are changing, depending on the mean direction of their neighbors within a specific radius; however, with angular noise (Ahmed I. S., 2018). A very non-trivial global behaviour can be the result of such a simple rule, where the system experiences the phase transition between disorder and order, when the noise is decreased or the particle density is increased (Ahmed I. L., 2017). The effects of speed variability, spatial confinement, anisotropy of the interactions, and heterogeneous media have been studied in variants of the Vicsek model (Vicsek, 1995).

The past research proved that the addition of noise interferes with the order, and the disorderly motion changes occur. Finite-size scaling and order parameters have been used to analyze this transition, as have correlation functions(Soomro et al., 2024). However, while much attention has been given to the disruption caused by noise, less is known about the system's ability to recover from such disruptions (Ansari et al., 2018). The knowledge of recovery is essential in the practicability of environments where noise and perturbation are inevitable and where the resilience of systems is important to ensure continued operations(Junejo et al., 2019).We discuss this gap, and in this paper, we focus on the disruption

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 3, Issue 5 (2025)

and recovery of collective motion in SPP systems subject to random noise. We want to determine important levels of noise-induced phase transitions and analyse the speed and efficacy with which the system restores coherence when the noise level is decreased. We also explore the role of particle density and exposure duration in the recovery process. Our work contributes to the broader understanding of dynamic phase transitions in active systems and offers insights relevant to both natural and engineered collectives.

MATHEMATICAL MODEL DESCRIPTION

We use a two-dimensional variant of the Vicsek model. Every particle is represented by its position $\vec{x}_i(t)$ and velocity direction $\theta_i(t)$. The position of the particles:

$$\vec{x}_i(t+1) = \vec{x}_i(t) + v_0 \vec{v}_i(t) \quad \text{(i)}$$

wherev_0 is the constant speed and $\vec{v}_i(t) = (\cos(\theta_i), \sin(\theta_i))$.

The direction θ_i is updated using the average direction of neighboring particles within a radius R, perturbed by angular noise:

$$\theta_i(t+1) = \langle \theta(t) \rangle_R + \eta \xi_i(t) \quad \text{(ii)}$$

Here, η is the noise amplitude and $\xi_i(t)$ is a variable which is random and uniformly distributed in $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

SIMULATION PARAMETERS

Number of particles $N = 1000$, Speed $v_0 = 0.03$, Interaction radius $R = 1$, Noise amplitude η varied from 0 to 1.0 in steps of 0.05, Box size =10, periodic boundary conditions are given in 2D, and Simulation time = 10,000 steps.

ORDER PARAMETER

Order parameter is used for the purpose to categorize collection of the SPPs

$$\Phi(t) = \frac{1}{N} \left| \sum_{i=1}^{N} \vec{v}_i(t) \right| \quad \text{(iii)}$$

RESULTS

DISRUPTION OF COLLECTIVE MOTION

At low noise levels , the order parameter remains close to 1, indicating strong alignment and coherence. As noise increases beyond a critical threshold (~0.5), sharply drops, signaling the transition to disordered motion. This critical point shifts slightly depending on the particle density.

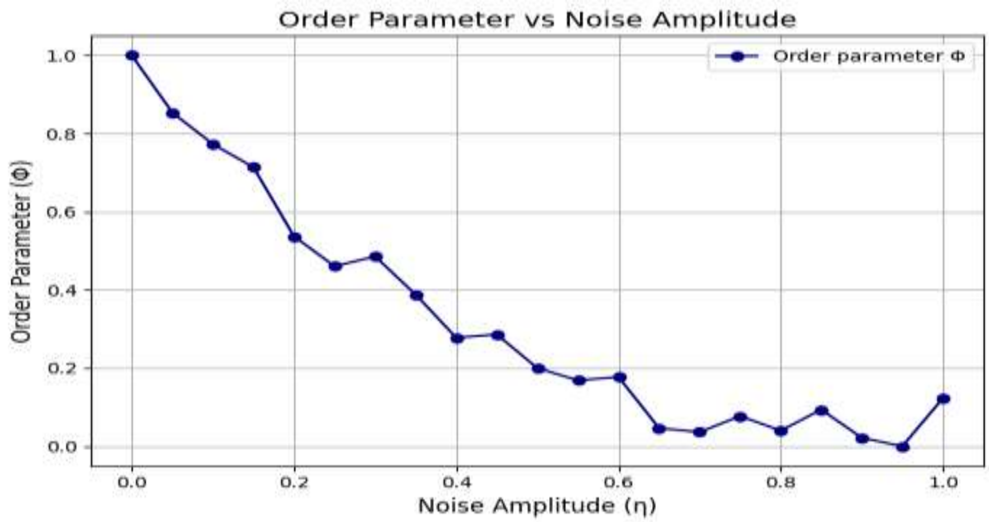

FIGURE 1. Collective motion as a function of noise

Order parameter Φ plotted against noise amplitude η. The results show a sharp decline in global alignment as noise increases, indicating a transition from ordered to disordered motion.

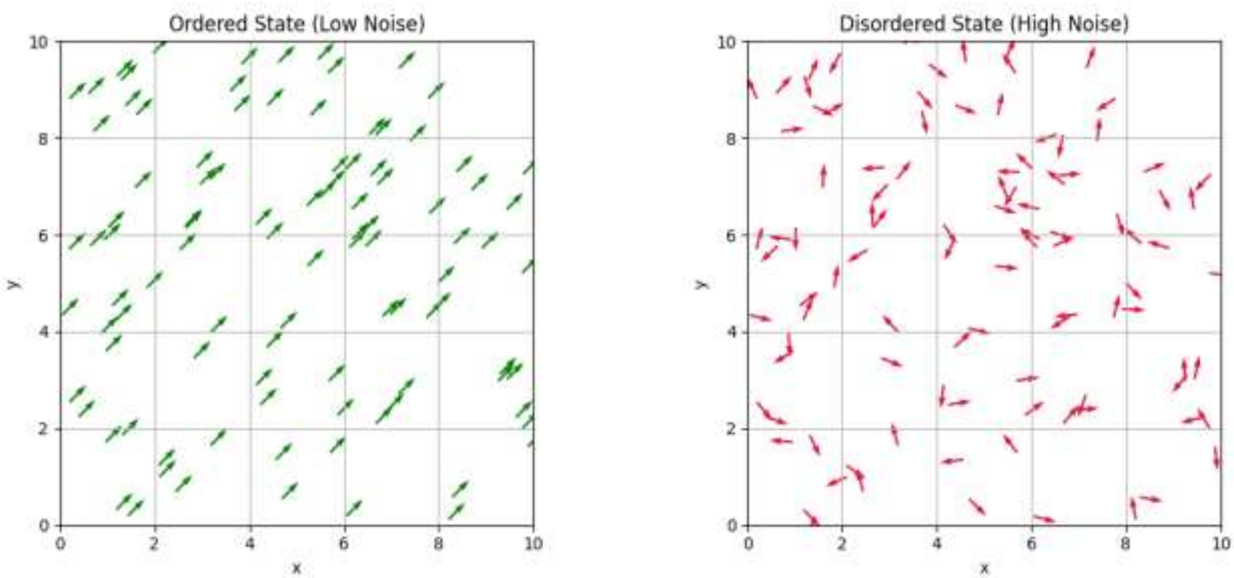

FIGURE 2. Demonstration of the collective motion of SPPs at different noise values.

The left figure 2 shows the ordered state with all particles aligned at low noise ($\eta = 0.1$), and the right figure2 shows the disordered state with random orientations at high noise ($\eta = 1.0$). These plots visualize the breakdown of collective motion due to noise.

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 3, Issue 5 (2025)

RECOVERY AFTER NOISE REDUCTION

When high noise (e.g., $\eta = 0.6$) is applied for a limited duration and then reduced back to $\eta = 0.1$, the system shows a gradual increase in Φ, indicating partial recovery. The recovery is faster and more complete at higher particle densities.

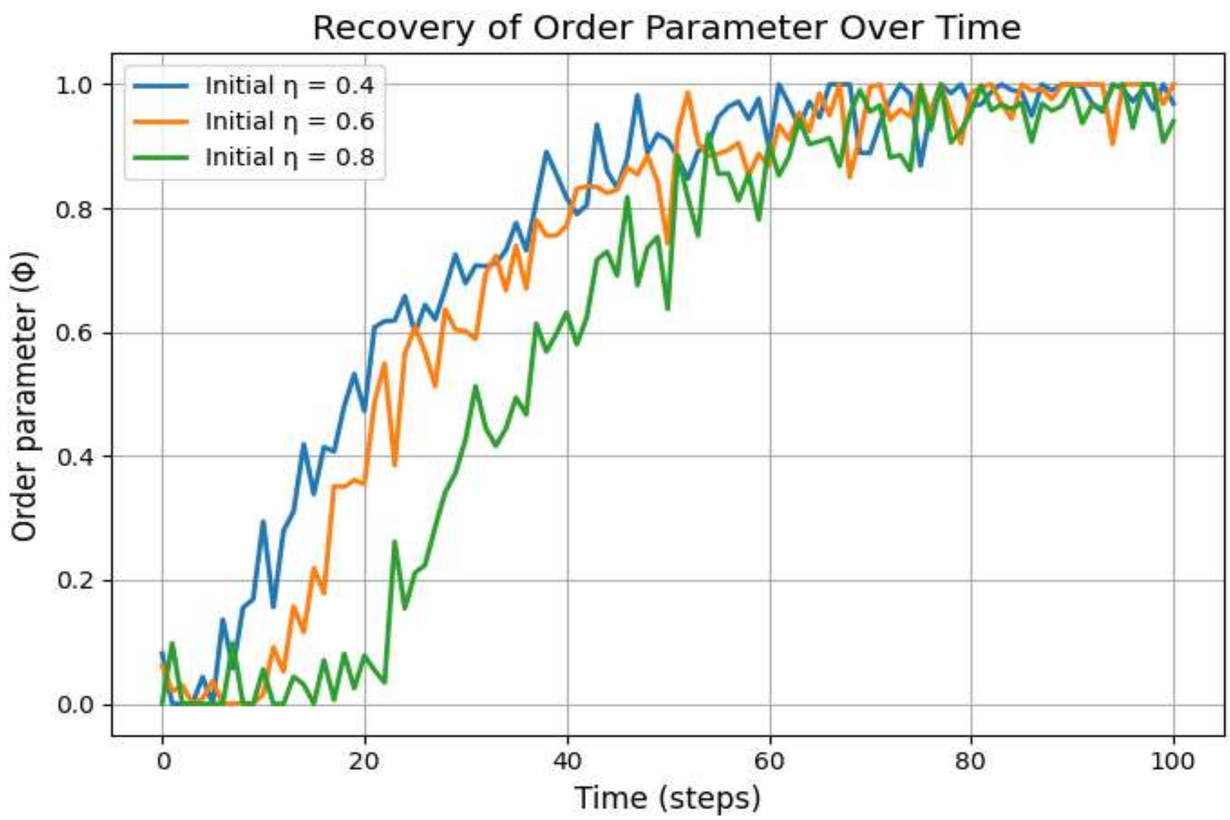

FIGURE 3. Time evolution of Φ following a reduction in noise. The recovery trajectory depends on the initial noise level, with higher η causing slower or incomplete recovery.

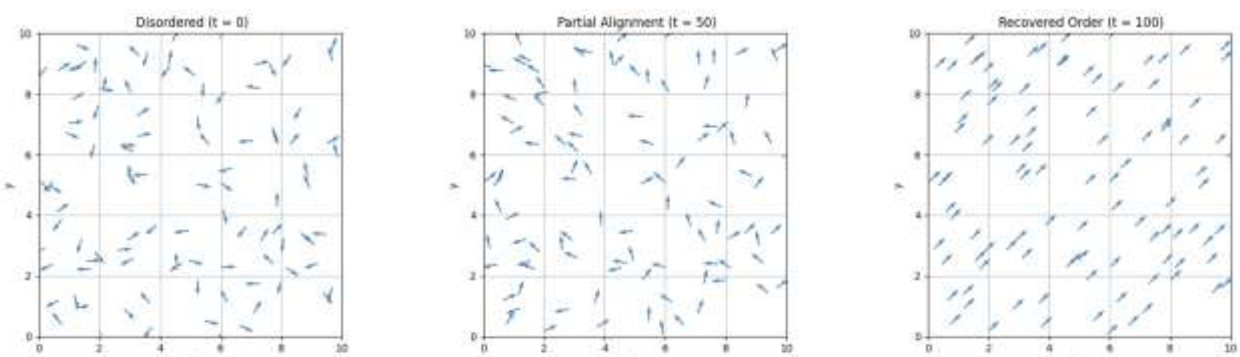

FIGURE 4. Snapshots showing directional alignment of particles at different recovery stages.At , particles are disordered; by , alignment is largely restored.

Annual Methodological Archive Research Review
http://amresearchreview.com/index.php/Journal/about
Volume 3, Issue 5 (2025)

Three noise values tested. All suggested similar kind of the behaviour at initial stage of the simulation where particle showed less collective motion. Soon after the 20th time step there appeared a rise in the value of the order parameter. At the 100th time step a higher collective motion found in the system which suggested a stability in the collective motion. This is evident in the figure 3 and figure 4.

PHASE DIAGRAM

We constructed a phase diagram mapping the steady-state Φ values over the (η,ρ) space. Three distinct regions emerge: fully ordered, disordered, and recoverable phases.

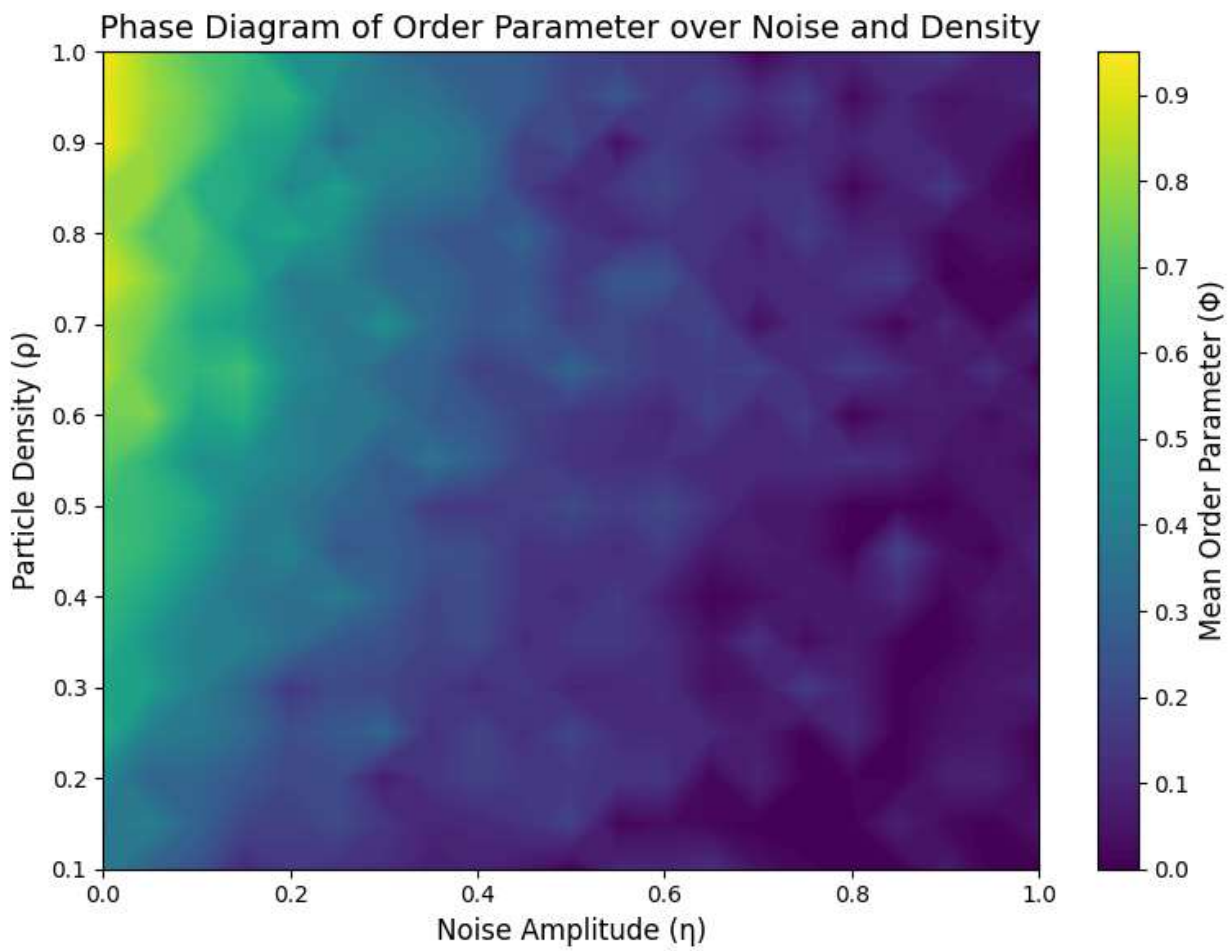

FIGURE 5. Phase diagram of order parameter over noise and density.

Phase map of the system over noise amplitude η and particle density ρ. The color scale represents the average order parameter Φ, revealing regions of ordered, disordered, and recoverable behavior.

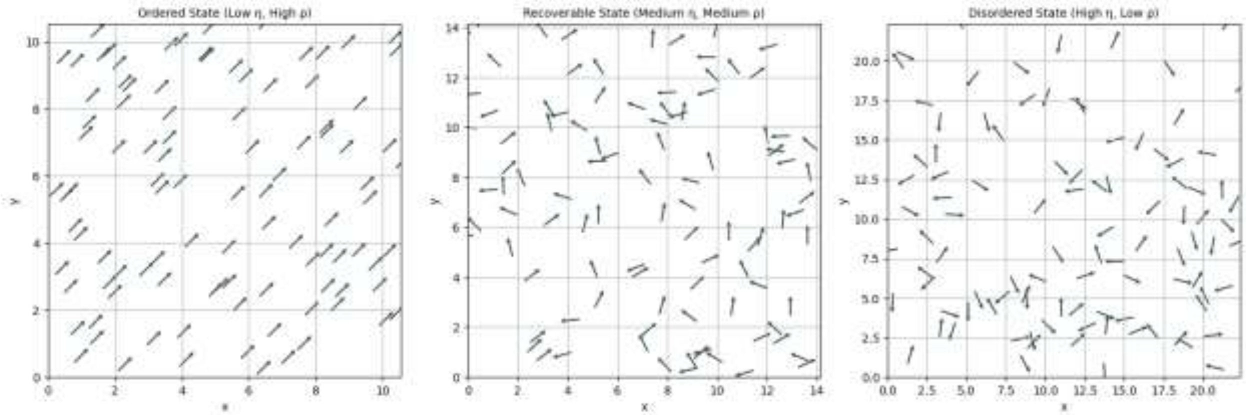

FIGURE 6. Arrow plots showing directional patterns in three regions of the phase diagram

CLUSTER DYNAMICS

Noise disrupts large clusters and results in more uniform distributions. Post-recovery, clusters reform if the system retains sufficient density and time to re-align.

FIGURE 7. Demonstration of cluster size distribution.

Histogram comparing cluster size distributions across ordered, recovering, and disordered phases. Ordered states show large clusters, while disordered states exhibit numerous small clusters.

DISCUSSION

our findings confirm the presence of a critical noise threshold beyond which collective order breaks down. This threshold demarcates a clear transition from coherent, collective motion to disordered, random behavior. The nature of this transition is critical and is particularly evident in the sharp decline of the order parameter as the noise increases to a specific value, which varies slightly depending on particle density(Zhang, 2021). This behavior is consistent with phase transitions observed in other non-equilibrium systems and supports the theoretical predictions made by earlier studies using the Vicsek model (Soomro I. A. I., 2019).

More importantly, we have demonstrated that the system's ability to recover coherence after noise reduction is not only possible but strongly dependent on both particle density and the duration of exposure to high noise. Higher densities facilitate more rapid and complete recovery, likely due to increased interaction frequency among particles. Conversely, prolonged exposure to strong noise can lead to persistent disordered configurations that take longer to reorganize, or may even fail to recover entirely(Chaté, 2008). These dynamics illustrate a form of hysteresis in active systems, where the path of recovery is not simply the reverse of disruption.Their simulation outcomes will have valuable information about the robustness of active matter systems. In ecology, this resilience may form the basis of how bird flocks, fish schools, or bacterial colonies can reorganize themselves in reaction to disturbances. In man-made systems (e.g. swarm robotics), the same findings open up the possibility that local communication and density control might be essential in re-establishing coordination after system-level failure or environmental noise.

On a design standpoint, the phase diagrams and cluster behavior studies provide handy guidelines on developing fault-tolerant systems capable of retaining or recovering performance against noisy backgrounds. As an example, recoverability might be guaranteed by keeping the particle density above some measure, even in the presence of either fluctuating or stochastic inputs. Also, re-align capacity implies that mechanisms can be enhanced by system-wide re-synchronization regimes or coordination polls.

CONCLUSION

We investigated in this paper how stochastic noise can interfere with and affect the collective-motion recovery of self-propelled particle systems. A Vicsek-type model helped us find a critical noise level that defines a transition between the disordered and ordered states. We also demonstrated that the recovery can occur under the condition of reduced noise, where particle density and exposure time are important factors that define the extent and rate of re-alignment.Order parameter analysis, phase behavior, and cluster distributions promoted distinctive regimes of system behavior: entirely ordered, disordered, and partially recoverable. The results help in gaining insights into the robustness of the active matter system and conditions under which self-organization becomes possible following perturbation. Our findings can be used in future studies and applied works with biological masses and engineered swarming systems, especially in a noisy or dynamic environment.

REFERENCES

Ahmed, I., Ly, D. Q., & Ahmed, W. (2017). Collective behavior of self-propelled particles in the presence of moving obstacles.. *Materials Today: Proceedings*, *4*(1), 65-74.

Ahmed, I., Soomro, I., Shah, S. B., & Shaikh, H. (2018).Simulation of collective motion of selfpropelled particles in homogeneous and heterogeneous medium. *International Journal of Computer Science and Network Security.*, *18*(11), 109.

Ali, M., Ahmed, I., Indher, A. M., & Shaikh, R. (2024). Speed Variability and Its Effects on the Mathematical Model of Self-Propelled Particles. *The Sciencetech*, *5*(4), 162-174.

Ansari, F. A., Ahmed, I., Dharejo, K. A., &Kalhoro, A. N. (2018).Computational and analytical solution of fractional order linear partial differential equations using Sumudu transform and its properties. *International Journal of Computer Science and Network Security.*, *18*(9), 43-50.

Chaté, H., Ginelli, F., Grégoire, G., & Raynaud, F. (2008).Collective motion of self-propelled particles interacting without cohesion. *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics*, *77*(4), 046113.

InayatullahSoomro, I. A., Shah, S. B., Majid, A., Muhammad, R., Hameed, A., Abas, G., ...& Ahmed, W. (2019). Mathematical modelling of cylindrical forming di-block copolymers confined in circular annular pores. *IJCSNS*, *19*(2), 16.

Jin, D., & Zhang, L. (2021). Collective behaviors of magnetic active matter: Recent progress toward reconfigurable, adaptive, and multifunctional swarming micro/nanorobots. *Accounts of Chemical Research*, *55*(1), 98-109.

Junejo, A. H., InayatullahSoomro, I. A., Kalhoro, A. N., & Muhammad, R. (2019). A connected graph with set of empty intersection of all longest cycles. *International Journal of Computer Science and Network Security*, *19*(5), 87.

Pismen, L. (2021). Active matter within and around us: from self-propelled particles to flocks and living forms.

Reynolds, A. M., McIvor, G. E., Thornton, A., Yang, P., & Ouellette, N. T. (2022). Stochastic modelling of bird flocks: accounting for the cohesiveness of collective motion. *Journal of the Royal Society Interface*, *19*(189), 20210745.

Soomro, F. A., Ahmed, I., Panhwar, W. A., &Soomro, S. R. (2024). Mathematical and computationalmodeling of migrations, population spread, and Location-dependent carrying capacities. *VFAST Transactions on Mathematics*, *12*(1), 248-263.

Vicsek, T., Czirók, A., Ben-Jacob, E., Cohen, I., &Shochet, O. (1995).Novel type of phase transition in a system of self-driven particles. *Physical review letters*, *75*(6), 1226.

Zhang, B. Q., & Shao, Z. G. (2021).Collective motion of self-propelled particles with complex noise environments. *Physica A: Statistical Mechanics and its Applications*, *563*, 125382.